Neighborhood Externalities and Residential Sorting

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November 25, 2020

Abstract

This paper studies the role of endogenous neighbourhood amenities in determining internal city structure, residential sorting, and house prices. The two key assumptions of the model are: (i) rich households have higher preference for city-type amenities; (ii) the amenity levels across locations in a city depends on the location of high-income households: living closer to the rich provides better access to amenities. The model can generate rich income-location relationships, depending on the interactions between endogenous amenities and commuting costs. When the income elasticity of commuting costs is relatively low, the model can generate polycentric city structure: endogenous amenities and low commuting costs together enable the formation of sub-centers. When the effects of endogenous amenities dominate, higher density near the city center facilitates provision of amenities, making the central locations more attractive for the rich.

Keywords: Residential Sorting, Monocentric Model, Neighborhood Externalities, assignment problem

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1 Introduction

The distribution of income across locations within cities is highly uneven, and there are large variations in the income-location relationships across cities and over time. In European cities, for example, rich households tend to live close to the city center. In America, cities were characterized by a long period of declining central cities and the "flight" of affluent residents to the suburbs (Baum-Snow 2007, Boustan 2010). During the last three decades, however, cities in North America have experienced a wave of "central city revival", during which well-educated, affluent residents moved to central neighborhoods. How households of different income levels distribute themselves within a city is central for various economic issues, such as traffic congestion, employment opportunities, and urban productivity. The impacts of local policies, such as housing subsidies, transport infrastructure improvement, zoning laws, crucially depend on how these policies affect household location choices.

The standard monocentric model (Alonso 1964, Mills 1984, and Muth 1969) provides insights into the effects of income on household location. However, the model fails to explains why affluent, well-educated residents are returning to city centres during the last decades, despite improvement in transit infrastructure and declining cost for automobiles. In this paper, I explore the fact that neighborhood amenities are an important consideration when households choose where to live within a city. Moreover, I allow for the fact that the amenities in a neighborhood are endogenous, and depend on the types of households that choose to live there. I formalize this idea is a spatial model of a city that features two income types and neighborhood-specific amenities.

In this paper I link the locations of different income groups within a city to endogenous neighborhood amenities. I consider a spatial model of a city with two income types. The city is represented by a line, and each household chooses a location within the city. Households have to travel to the city center for work and to access exogenous amenities, and traveling to the city center incurs income-dependent time costs. Exogenous amenities, such as historical monuments, main railway station, and natural amenities are fixed at the city center. A primary difference from a standard monocentric city model is that households also derive utility from neighborhood-specific amenities, which depend on the location of high-income households within the city.

The key assumption of the model is that households have non-homothetic preferences for neighborhood amenities. It is assumed that each household's utility is increasing in their proximity to the amenities that are concentrated in the neighbourhoods where high-income households reside. Moreover, access to and therefore the utility derived from these amenities is assumed to increase with income level. Although I do not explicitly micro-found the mechanism, there are many potential channels through which households benefit from living close to those with high incomes. For example, if there are increasing returns to scale in the production of neighborhood amenities (number and variety of restaurants, easier access to services), such amenities may be more common as the income of one's neighbors increases. The operation of venues like restaurants and cinemas crucially depends on high population density to allow access to a large customer base. Living closer to the rich, who gain higher utility from these amenities, means better access to these amenities.

In this paper I demonstrate how such externalities can generate the indeterminacy in income-location relationship in a spatial model of a city. I show that different types of equilibria with income segregation may arise, depending on the interactions between commuting costs and preferences for neighborhood-specific amenities. Endogenous neighborhood amenities may reinforce the rich households' tendency to concentrate near the center, or foster the formation of rich neighborhoods in the suburbs, depending on the size of the income elasticity of commuting costs. In particular, when the income elasticity of commuting costs is high, rich households concentrate near the city center, and neighborhood-specific amenities reinforce this tendency. When the income elasticity of commuting costs is low, the equilibrium income-location relationship is indeterminate; the rich can either live in the central city or the suburbs.

The model sheds light on the causes of suburbanization and neighborhood gentrification. The equilibrium where the rich segregate in the suburbs can only exist if the income elasticity of commuting costs is low enough, implying that decreasing commuting costs is necessary for suburbanization to take place. When the preference for externalities is non-homothetic, however, the equilibria where the rich move to centres of cities can arise even for low commuting costs. Because the high density of the central city increases the scale of externalities, rich households, who benefit more from them, would optimally congregate there, even commuting from the suburbs is not costly.

My work contributes to the literature on endogenous amenities and residential sorting in a city. Brueckner et al. (1997) propose an amenity-based theory that ties the location of income to the distribution of exogenous amenities. My model can be thought of as an extension of their amenity-based model, where exogenous amenities are located in the city center, and neighborhood-specific amenities are endogenously provided in the rich neighborhoods. Lee and Lin (2018) consider a model with both exogenous and endogenous amenities, and find that superior natural amenities, like beaches, lake views, anchor the locations of rich neighborhoods within cities. My model shows that non-homotheric preferences for neighborhood-specific amenities increase high-income households' tendency to concentrate near the city center, where exogenous amenities locate. The theoretical model and solution strategy in this paper is similar to Guerrieri et al. (2013), where neighborhood externalities cause poor neighborhoods adjacent to rich ones to gentrify during city-wide housing booms. Here, however, I incorporate spatial aspects that induce land rents to decline with distance from the city centre.

paper complements works on non-homothetic preferences for local amenities. Using product level data, Handbury et al. (2015) finds evidence for non-homothetic preferences for local consumption goods. Diamond (2016) shows that local amenity values endogenously improve as the city is populated by more well-educated, high-skilled workers. My work is most closely related to Couture (2013), which finds that the rising taste of well-educated, young residents for non-tradable service amenities accounts for more than 40 percent of their moving to city centers. In this paper, I highlight the role of non-homothetic preferences for neighborhood externalities in explaining the cross-city differences in income-location relationships.

My model also sheds light on the causes of suburbanization. One strand of literature emphasizes the role of transportation innovations and travel time considerations. For example, Baum-Snow (2007) shows that new highways are an important element to explain urban population decentralization in a monocentric city model. Another strand of the literature highlights fiscal and social problems of central cities. Problems like low quality public schools, and high crime rates lead rich residents to migrate to the suburbs, forming homogeneous communities.(Mills and Price (1984)) My framework combines these two theories together by explicitly modeling both commuting cost and non-homothetic preferences for endogenous amenities.

My work belongs to the emerging literature on the causes of the recent "central city revival". For example, Ellen et al. (2019) study the role of reduced crime rates in central cities. Su (2018) and Edlund et al. (2015) highlight the demand for shorter commutes by high-skilled workers. My mechanism is mostly related to Couture and Handbury (2017) and Couture et al. (2019), in which well-educated, young residents' non-homothetic preference for city-type amenities increase their demand for central locations. In my paper, I study the interactions between commuting costs and non-homothetic preference for endogenous amenities in a monocentric city model. My results show that with non-homothetic preferences for neighborhood amenities, the equilibrium where the rich concentrate near the city center is more likely to arise, indicating that non-homothetic preferences for neighborhood externalities is a key driving force for gentrification of the central cities.

The rest of this paper is organized as follows. In the next section, I develop a model of residential sorting and construct two types of equilibria with different income-location patterns. Then I solve for the constructed equilibria numerically and explore the effects of externalities on equilibrium outcomes. The last section concludes and discusses future work.

2 The Model

Consider a city populated by N households comprised of two types: a continuum of rich households of measure N_R and a continuum of poor households of measure N_P. Households of type s, $s \in \{R, P\}$, receive exogenous income endowment y_s, with $y_P < y_R$.

The city is represented by the real line and each point on the line $x \in (-\infty, +\infty)$ is a different location. Agents are fully mobile and can choose to live in any location x. Point $x = 0$ is normalized as the center of the city. All residents have to travel to the city center $x = 0$ for work and to access exogenous amenities. Traveling incurs a pecuniary cost $T_s(x) = \tau_s x$, $s \in \{R, P\}$. It is assumed that the marginal commuting cost is higher for the rich, i.e., $\tau_R > \tau_P$, because the opportunity cost to travel increases income.

The key assumption of the model is that there are positive location externalities: households benefit from living closer to rich households, whose presence at a location increases the amenity level. In this paper, I focus on equilibria with full income segregation, so a location is occupied either by the rich or the poor. Denote $\mathbb{1}(x)$ as an indicator function which equals 1 if location x is inhabited by rich households and equals 0 otherwise. The effect of rich residents in location x on other households decays exponentially with the distance between them. Let $E(x)$ denote the total value of amenities experienced by a household in location x. Denote the boundary of the city as b. Denote the distance between x and a location s. The amenity value $E(x)$ takes the following functional form:

$$E(x) = \int_{-b}^{b} e^{-\gamma|x-m|} \mathbb{1}(m) dm, \; \forall m \in [-b, b], \tag{1}$$

where $|x-m|$ is the distance between location x and location m, which is occupied by high-income households. Note that this specification implicitly assumes that externalities increase with the density of rich households: externalities are higher if rich household live closer to each other. The effects of higher urban density on consumers is addressed by Glaeser and Gottlieb (2006)[1] and Couture (2013)[2]. In this model, these externalities are captured by the fact that households enjoy their houses more if they live in locations with higher externalities.

Households derive utility from consumption good c and housing services $H(x)$. Assume that utility takes the following functional form: $u(c, H) = c^{1-\beta} H^{\beta}$, for $s \in \{R, P\}$. Housing service $H_s(x) = h(x)E(x)^{\delta_s}$, which depends on housing units consumed and the level of externalities. It is assumed that $\delta_R \geq \delta_P$, so that rich households, who generate the externalities, value them at least as much as poor households. In reality, certain public goods or services, like high-quality schools, are financed locally and are more accessible to the rich. Some amenities at the neighborhood level, such as fancy restaurants and expensive entertainments, are

[1]Glaeser and Gottlieb (2006) point out that higher density improves consumers' access to a greater variety of goods and services.

[2]Couture (2013) shows that increased density enables consumers to gain access to more varieties and save time through shorter commute.

more specific to high income residents.

On the supply side, there is a perfectly competitive construction industry using land and capital to produce housing under constant returns to scale technology. Construction firms produce $f(x)$ units of housing services per unit of land at x. Denote the rental price of land as $R(x)$. Assume that the rental price of capital is constant and exogenously given, so it can be omitted from the unit cost function $C(R(x), 1)$. Denote the price per unit of housing as $P(x)$. At each location, the construction firms make zero profits, so $P(x) = c(R(x))$. Denote $\underline{R}$ the value for best alternative land use. Land will be developed whenever $R(x) > \underline{R}$.

2.1 The Bid-rent Function

$P_s(x, u_s)$, the bid-rent function, is defined as the highest price that households of type s are willing to pay to reside at location x give utility level u_s while satisfying the budget constraint:

$$P_s(x, u_s) \equiv \max_{c^s(x), h^s(x)} \{P(x) | u(h, c) = u_s, c^s(x) + h^s(x)P(x) \leq y^s - \tau_s x\},\ s \in \{R, P\}. \tag{2}$$

Let us replace $c(x)$ with the restricted Hicksian demand for consumption good $c(h(x), u^s)$, and solve for $P(x)$ from the budget constrain. The maximization problem becomes

$$P_s(x, u^s) = \max_{h^s(x)} \Big\{ \frac{y^s - \tau_s x - c(h^s(x), u^s)}{h^s(x)} \Big\}. \tag{3}$$

From the utility function, we can obtain $c\left(h^s(x), E(x), u_s\right)$. Substituting this into the above expression, and solving for $h^s(x)$ yields

$$h^s(x) = \left[\frac{u_s E(x)^{-\delta_s \beta}}{(1-\beta)^{1-\beta}(y_s - \tau_s x)^{1-\beta}}\right]^{\frac{1}{\beta}} \tag{4}$$

$$P_s(x, u_s) = \max_{h^s(x)} \left\{ \frac{y^s - \tau_s x - h^{-\frac{\beta}{1-\beta}} u^{\frac{1}{1-\beta}} E^{-\frac{\delta^s \beta}{1-\beta}}}{h^s(x)} \right\}. \tag{5}$$

Reinserting $h^s(x)$ into the maximization problem yields

$$P_s(x, u_s) = (1-\beta)^{\frac{1-\beta}{\beta}} \beta \left(\frac{1}{u^s}\right)^{\frac{1}{\beta}} \left[y^s - \tau_s x\right]^{\frac{1}{\beta}} E(x)^{\delta^s}, \ s \in \{R, P\} \tag{6}$$

Given the externalities and commuting costs , a household of type s is willing to pay $P_s(x)$ for one unit of housing at location x, such that a common utility level u_s can be obtained across locations that he chooses. This expression is intuitive, as households are willing to pay higher unit prices for housing at locations which are closer to the city center, or where neighborhood-specific amenities are higher.

The unit house price at location x is the upper envelop of two bid-rent functions, that is

$$P(x) = \max\{P_R(x, u_R), P_P(x, u_P)\}. \tag{7}$$

2.2 Supply Side

At each location x, a representative construction firm chooses the amount of land $l(x)$ and capital $k(x)$ needed to produce housing, taking as given the rental price of land $R(x)$. Assuming a Cobb-Douglous production function for construction firms, we can define the unit cost function for the firms as

$$\begin{aligned} c(R(x),1) = & \min_{l(x),k(x)} \quad R(x)l(x) + \bar{r}k(x), \\ \text{subject to} & \qquad l(x)^{\alpha}k(x)^{1-\alpha} \geq 1. \end{aligned} \tag{8}$$

From the above cost minimization problem, we can obtain the unit cost to produce one unit of housing service:

$$c(R(x),1) = A * R(x)^{\alpha}, \tag{9}$$

where $A = \bar{r}^{1-\alpha}\frac{(1-\alpha)^{\alpha-1}}{\alpha^{\alpha}}$. The cost of building one unit of housing is higher if land is more expensive.

By Shepard's Lemma, the Hicksian demand for land needed to produce one unit of housing is given by

$$\frac{\partial c(R(x))}{\partial R(x)} = \frac{1}{f(x)} = \alpha A R(x)^{\alpha-1}. \tag{10}$$

At each location, there is one unit of land. So the total units of housing produced at x is

$$f(x) = \frac{1}{\alpha A} R(x)^{1-\alpha}.$$

Housing supply $f(x)$ is increasing in $R(x)$: more housing units will be produced if the rental price for land is higher at a location.

The measure of households, or population density at x, is given by

$$n(x) = \frac{f(x)}{h(x)} = \frac{1}{\alpha\beta} \left[\frac{1}{y_s - T(y_s, x)} \right] R(x). \tag{11}$$

Because of free entry, construction firms make zero profit at each location, so the unit price for housing should equal the unit cost

$$P(x) = c(R(x)) = AR(x)^{\alpha}. \tag{12}$$

The zero profit condition establishes a one-to-one relationship between the unit housing price $P(x)$ and the rental price of land $R(x)$. Unit housing prices are higher where land is more expensive.

Market Clearing Condition Denote by $n_s(x)$ the measure of households of type s who live in location x and by $h_s(x)$ the number of housing units, or the size of the house they choose. Market clearing requires

$$\int_{-\infty}^{+\infty} n_s(x) dx = N_s, \; s \in \{R, P\}. \tag{13}$$

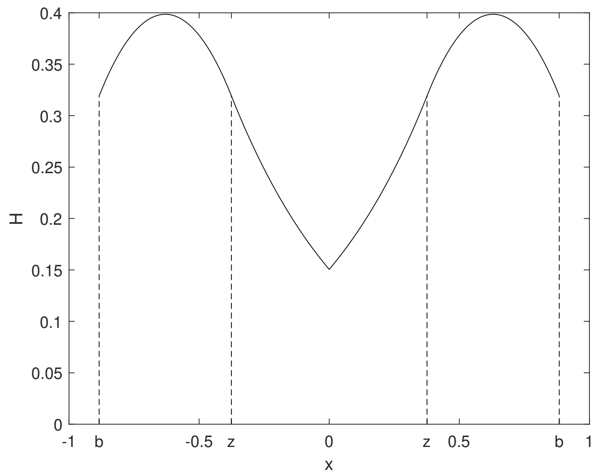
Equilibrium Definition An equilibrium is a sequence of price schedules $\{P(x), R(x)\}_{x\in R}$, and allocations $\{n_R(x), n_P(x), h_R(x), h_P(x)\}_{x\in R}$, such that households maximize utility, the construction sector minimizes costs and makes production zero profits, and markets clear conditions 13 are satisfied.

3 Equilibrium Construction

With income and location heterogeneity, the sorting problem is complicated and many types of equilibrium may exist. In this paper, I focus on two types of equilibrium with full separation. I first consider a "suburbanized equilibrium" where the poor concentrate in the city center and the rich live at the periphery. The locational pattern is reversed in the "gentrified equilibrium" where the rich households live in the city center. For each case, I first derive the conditions needed to solve the model under the equilibrium definition. Then I check that the solution is indeed an equilibrium by showing the that willingness to pay is higher for the type of households in their "proposed" neighborhood. In this section, I will only derive the equilibrium conditions to solve the "suburbanized equilibrium". The conditions for the "gentrified equilibrium" can be found in the Appendix D.

Denote by z the point that separates both income types. Both z and the city edge b are equilibrium objects. Given that the proposed equilibrium is symmetric in x, from now on, I restrict attention to $x \geq 0$.

Figure 1: The "Suburbanized Equilibrium": Amenity Level across Locations



Note: This figure shows the externalities across locations within the city when the equilibrium is such that the rich in the suburbs. In this example, the boundary for the poor and rich neighborhood $z = 0.3578$, and city size $b = 0.8842$.

3.1 The "Suburbanized Equilibrium"

Let us first consider the equilibrium where the poor households live in locations $x \in [0, z)$, and the rich live in locations $x \in (z, b]$. Under this equilibrium configuration, the externality function (1) can be derived as:

$$E(x) = \begin{cases} \frac{e^{-\gamma(z-x)} - e^{-\gamma(b-x)}}{\gamma}, & x \in [0, z) \\ \frac{2 - e^{-\gamma(x-z)} - e^{-\gamma(b-x)}}{\gamma}, & x \in [z, b] \end{cases} \tag{14}$$

Figure 1 shows the value derived from amenities $E(x)$ as a function of location x when the equilibrium is such that the rich live in the suburbs. Externalities first increase with distance from the city center $x = 0$, reach the maximum at the center of the rich neighborhood segments $x = \pm\frac{z+b}{2}$, and then decrease as we approach the city boundary b.

Denote $R_s(x)$ as the rental price at location x where type s households reside. Households of type s are willing to pay $P_s(x)$ for one unit of housing at that location. From the price function (6) and zero profit condition (12), $R_s(x)$ can be written as

$$R_s(x) = K_s\left[y_s - T(y_s, x)\right]^{\frac{1}{\alpha\beta}} E(x)^{\frac{\delta_s}{\alpha}},\ s \in \{R, P\}. \tag{15}$$

K_s, with $s \in \{R, P\}$, depend on the equilibrium utility level u_s as well as several other exogenous parameters, and will be solved as an equilibrium object.

Rental price for land at the boundary of the city b is equal to its value for alternative land use, $\underline{R}$:

$$R_R(b) = K_R\left[y_R - T(y_R, b)\right]^{\frac{1}{\alpha\beta}} E(b)^{\frac{\delta_R}{\alpha}} = \underline{R}.$$

So K_R can be written as

$$K_R = \underline{R}\left[y_R - T(y_R, b)\right]^{-\frac{1}{\alpha\beta}} E(b)^{-\frac{\delta_R}{\alpha}}. \tag{16}$$

At $x = z$ which separates the two groups, the rental price must be the same for both types:

$$R_R(z) = R_P(z).$$

From the expression for the rental price (15), the above expression can be written as

$$K_R \left[y_R - T(y_R, z)\right]^{\frac{1}{\alpha\beta}} E(z)^{\frac{\delta_R}{\alpha}} = K_P \left[y_P - T(y_P, z)\right]^{\frac{1}{\alpha\beta}} E(z)^{\frac{\delta_P}{\alpha}}.$$

Plugging in K_R gives

$$K_P = K_R \left[\frac{y_R - T(y_R, z)}{y_P - T(y_P, z)}\right]^{\frac{1}{\alpha\beta}} E(z)^{\frac{\delta_R - \delta_P}{\alpha}} \tag{17}$$

Combining the measure of household (11) and the unit price (6), the market clearing conditions can then be written as

$$\int_0^z n_P(x)dx = \frac{1}{\alpha\beta} K_P \int_0^z \left[y_p - T(y_P, x)\right]^{\frac{1}{\alpha\beta}-1} E(x)^{\frac{\delta_P}{\alpha}} dx = \frac{1}{2} N_P, \tag{18}$$

$$\int_z^b n_R(x)dx = \frac{1}{\alpha\beta} K_R \int_z^b \left[y_R - T(y_R, x)\right]^{\frac{1}{\alpha\beta}-1} E(x)^{\frac{\delta_R}{\alpha}} dx = \frac{1}{2} N_R. \tag{19}$$

The solution to the model is represented by the four equations (16) to (19). From these four equations we can solve for K_P, K_R, the neighborhood boundary z, and the city boundary b. After obtaining z and b, we must verify that the solution is indeed an equilibrium by checking that the rich are willing to pay more than the poor to live in the rich neighborhood, and vice versa. Specifically,

$$P_R(x) \leq P_P(x) \quad \text{for all } x \in (0, z],$$

$$P_R(x) > P_P(x) \quad \text{for all } x \in (z, b].$$

3.2 The Effects of Externalities on Households' Locations

Proposition 1. *When $\delta_R = \delta_P$, neighborhood externalities do not affect the relative location of the rich and poor. The "gentrified equilibrium" exists if and only if* $\frac{T_x'(y_R,z)}{h_R(z)} > \frac{T_x'(y_P,z)}{h_P(z)}$. *The "suburbanized equilibrium" if and only if* $\frac{T_x'(y_R,z)}{h_R(z)} < \frac{T_x'(y_P,z)}{h_P(z)}$.

This proposition proves that neighborhood externalities do not affect the relative location of households within the city when both income types have the same preference for neighborhood externalities. When $\delta_R = \delta_P$, the relative location of the rich and the poor depends on the relative strength of the income elasticity of marginal commuting costs and the income elasticity of housing demand at the neighborhood boundary z, which is consistent with the prediction of the canonical monocentric city model without neighborhood externalities. When $\delta_R \neq \delta_P$, it is hard to determine the relative location of two income types analytically. I will explored the equilibrium configurations numerically when the preferences for externalities differ by income levels in Section 4.3.

4 Numerical Examples

In this section, I first solve for the "suburbanized equilibrium" and the "gentrified equilibrium" numerically. Then I explore how the equilibrium income-location relationship varies with commuting costs and preferences for neighborhood amenities. Specifically, I study which type of equilibrium exists when the values for τ_R and δ_R are varied, holding everything else constant. The model is solved under the parameter values in Table 1.

Table 1: Parameter Values for Numerical Examples

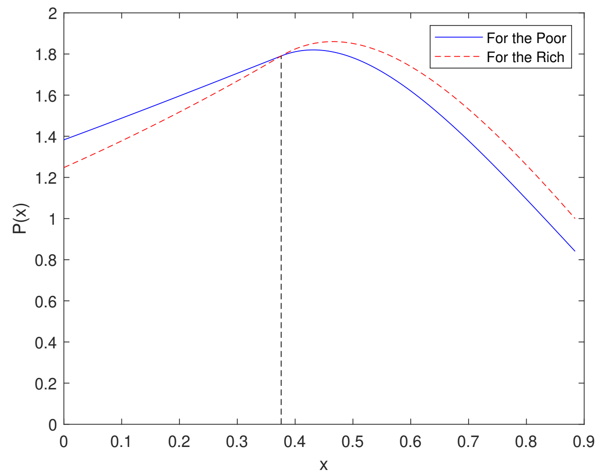
Parameter symbol	Definition	Value
α	expenditure share of land	0.5
β	income share of housing	0.2
$\underline{R}$	rent for alternative land use	1
τ_R	marginal commuting cost for the rich	0.6
τ_P	marginal commuting cost for the poor	0.5
y_R	income for the rich	1.5
y_P	income for the poor	1
γ	decay rate for externality	3
δ_R	the rich's preference for externality	1
δ_P	the poor's preference for externality	1
N_t^R	measure of rich household	10
N_t^P	measure of poor household	10

4.1 The "Suburbanized Equilibrium"

In this subsection, I solve for the "suburbanized equilibrium" under the specified parameter values. I first obtain the city boundary b and neighborhood boundary z from Equation (18) and (19). Then we need to verify that the solution is indeed an equilibrium. Figure 2 plots the willingness to pay for the rich and poor across locations under the "suburbanized equilibrium". The willingness to pay for the rich is higher than that for the poor in the suburbs, and the relationship is reversed in the city center. This is consistent with the definition of the proposed equilibrium . Thus the solution to Equation (18) and (19) is indeed an equilibrium.

After obtaining b and z, we can compute other equilibrium objects. The top panel of Figure 3 shows the unit housing price $P(x)$, housing size $h(x)$, total housing

Figure 2: The “Suburbanized Equilibrium”: Household’s Willingness to Pay



Note: Willingness to pay for the rich and poor when $\tau_R = 0.6$*, and the equilibrium is such that the rich live in the suburbs.*

expenditure $h(x)n(x)$, and population density $n(x)$ across locations in the city. In this equilibrium, commuting costs increase with distance, but neighborhood-specific externalities decline from the center of the rich neighborhood. The price gradient is hump-shaped, reflecting the fact that scale of amenities reaches a maximum at the center of the suburban neighborhood. In the poor neighborhood, the unit housing price P(x) increases as we move away from the city center, reflecting the fact that the benefit from living closer to the rich neighborhoods dominates that of locating centrally. Poor households are willing to live in smaller houses at locations adjacent to the rich neighborhood, where amenities are higher than other locations in the poor neighborhood. High externalities in the middle of the rich neighborhood cause households to concentrate near the neighborhood boundary z, where commuting costs are moderate and externalities are high.

Figure 3: The "Suburbanized Equilibrium": Numerical Solution

Note: Solution to the model when $\tau_R = 0.6$, and the equilibrium is such that the rich live in the suburb.

Note: Solution to the model when there is no neighborhood externalities and the equilibrium is such that the rich live in the suburb.

To see the effects of neighborhood externalities on the equilibrium outcomes, I compute the equilibrium for the standard monocentric model without externalities by setting $\delta_R = \delta_P = 0$. The bottom panel Figure 3 presents the equilibrium gradients for the model without externalities. Gradients in each neighborhood are monotonic, solely driven by linear commuting costs and the demand for housing. Rich households live in larger dwellings in the suburbs, where unit prices are lower than in the poor neighborhood. Without externalities, the size of the poor neighborhood is a lot smaller, because living further from the city center only incurs commuting costs.

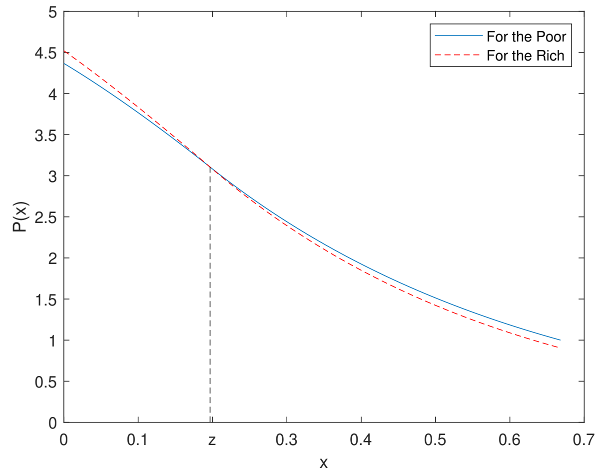
The equilibrium configuration with endogenous amenities in Figure 3 can be interpreted as a polycentric city. Although jobs and exogenous amenities are concentrated at the city center, endogenous neighborhood amenities in the suburbs where the rich households live lead to sub-centers in the middle of the city.[3] This equilibrium configuration arises for low income elasticity of commuting costs, indicating that the polycentric city structure is more common for cities with low commuting costs, or low value for exogenous amenities near the centre.

4.2 The "Gentrified Equilibrium"

In this subsection, I increase τ_R so that the rich would live near the city center in equilibrium. Figure 10 shows the willingness to pay for both income types across locations. The rich are willing to pay more to live in the central city, consistent with

[3]Fujita and Ogawa (1982) considers an urban land use with agglomeration in productions. In their model, both the locations of firms and households are endogenous. The model can generate cities that are either monocentric of polycentric, depending on the location choice of firms in the city. In this chapter, I show that endogenous externalities can lead to sub-centers for consumption under low income elasticity of commuting costs.

Figure 4: The "Gentrified Equilibrium": Household's Willingness to Pay

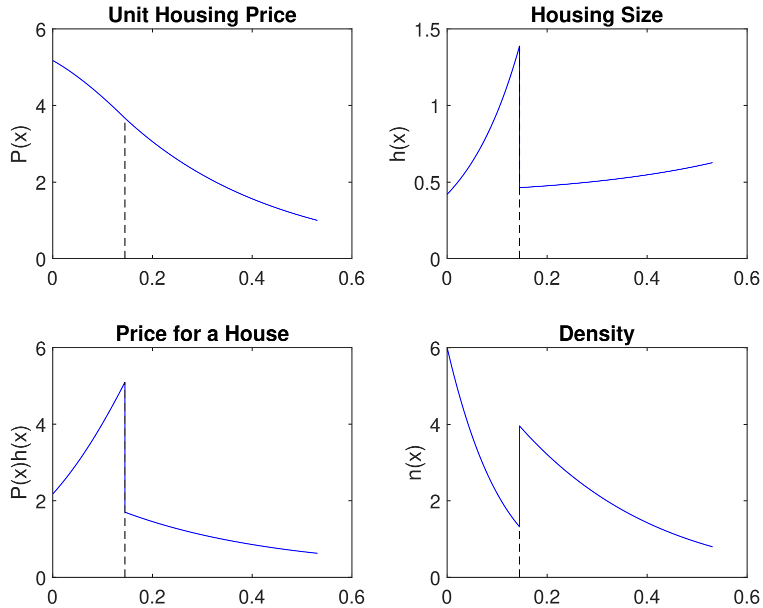


Note: Willingness to pay for the rich and the poor when $\tau_R = 0.9$, and the equilibrium is such that the rich live near the center.

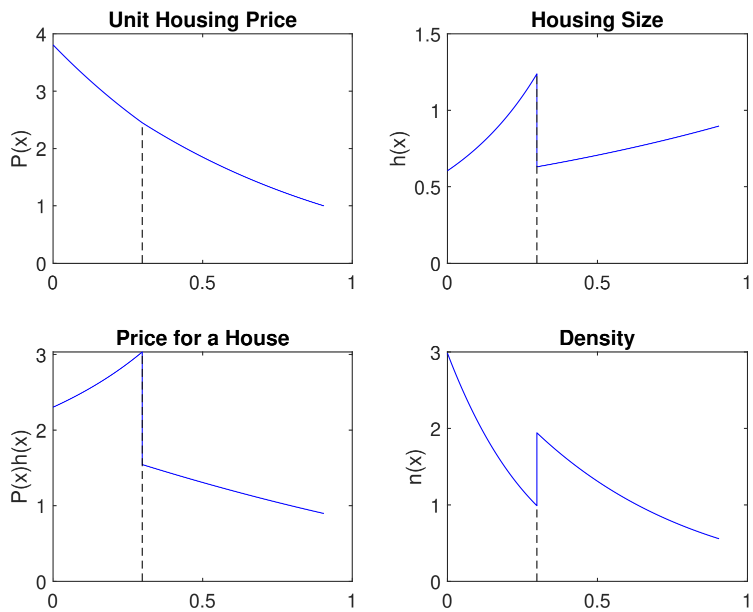
the definition of the proposed equilibrium. Such an equilibrium is sustained by the fact that when commuting is expensive for the rich, they would like to live closer to the city center, and externalities reinforce this tendency.

Figure 5 shows the numerical solution to the "gentrified equilibrium". Locations further from the center becomes less attractive, as commuting costs increase and externalities decay with distance. In the rich neighborhood, price declines as we move further from the city center, and housing size becomes larger as land gets cheaper. Total housing expenditure increases with distance, since the unit price declines at a slower rate than housing size increases. Population density follows the same pattern as unit price. In the poor neighborhood, unit price declines and housing size increases approaching the city boundary. Total housing expenditure declines, as the effect of declining unit house prices dominates. Population density declines as

Figure 5: The "Gentrified Equilibrium": Numerical Solution



Note: Solution to the model when $\tau_R = 0.9$, and the equilibrium is such that the rich live near the city center.



Note: Solution to the model when $\tau_R = 0.9$, and there is no neighborhood externalities.

we move closer to the city boundary.

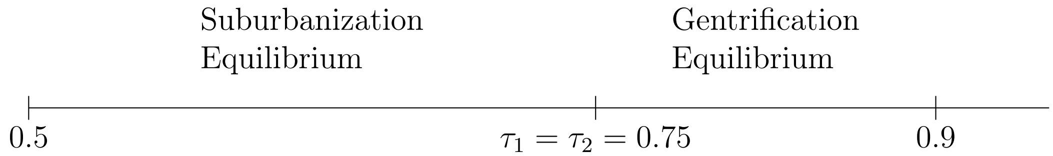
For comparison, I solve the model under the equilibrium configuration where the rich live near the city center without externalities. The equilibrium gradients can found at the bottom panel of Figure 5. In the "gentrified equilibrium" with externalities, the effects of externalities and commuting costs work in the same direction. As a result, both the city as a whole and the rich neighborhood become more compact compared with then there are no externalities. The gradients with and without externalities exhibit the same patterns. Prices and population density are higher with externalities, especially in the rich neighborhood.

4.3 Externalities, Commuting Costs, and Equilibrium Residential Patterns

In the previous examples, I generate a "suburbanized equilibrium" and a "gentrified equilibrium" by solving the model for a fixed set of parameter values. To further explore how the interactions between commuting costs and neighborhood externalities affect the relative location of the rich and the poor, I investigate which type of equilibrium arises for different values of τ_R and δ_R.

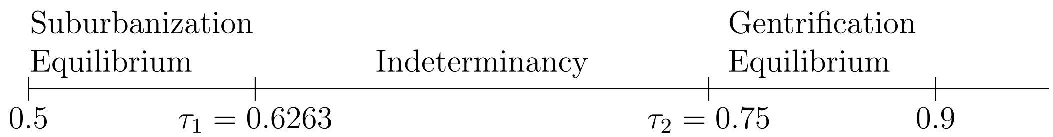
I first assume that the preference for neighborhood externalities is the same for two income groups, i.e., $\delta_R = \delta_P$, and look at which type of equilibrium configuration arises for different values of $\tau_R \in [0.5, 0.9]$. Figure 6 shows the equilibrium location of both households for different values of τ_R. In this case, there is no indeterminacy in the equilibrium income-location relationship. Either the rich or the poor live near

Figure 6: Interactions Between Commuting Costs and Externalities: $\delta_R = \delta_P$



Note: Equilibrium income-location relationships under different values of τ_R. In this example, $\delta_R = \delta_P = 0.2$.

Figure 7: Interactions Between Commuting Costs and Externalities: $\delta_R > \delta_P$

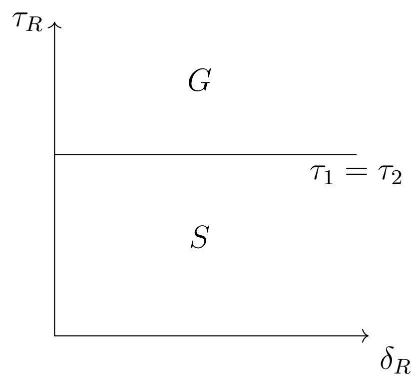


Note: Equilibrium income-location relationships under different values of τ_R. In this example, $\delta_R = 0.25$ and $\delta_P = 0.2$.

the city center, depending on the value of τ_R. Externalities do not affect the relative location of the rich and the poor, thus confirming Proposition 1.

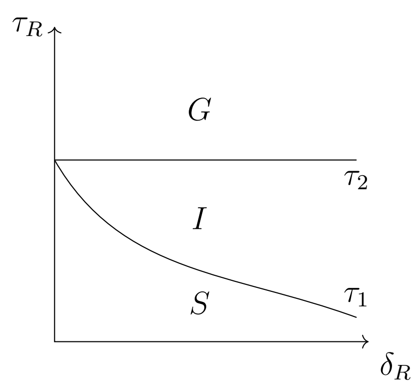
To see the effects of neighborhood externalities, I set $\delta_R > \delta_P$, and compute which type of equilibrium exists for different values of $\tau_R \in [0.5, 0.9]$. Figure 7 shows the equilibrium configurations for different values of τ_R. We can see that there are two cutoff values for τ_R, such that when $\tau_R \in [\tau_1, \tau_2]$, the equilibrium income-location relationship is indeterminate; when $\tau_R < \tau_1$, the unique equilibrium is the "suburbanized equilibrium" where the rich live in the suburbs; when $\tau_R > \tau_2$, the unique equilibrium is always the rich choose to live in the city center. Although low commuting costs foster suburbanization, for medium values of the income elasticity of commuting costs, the model predicts that the rich still prefer to live close to the city center, because externalities increase the attractiveness of central locations.

Figure 8: Homothetic Preference for Externalities and Equilibrium Residential Patterns



Notes: Relative location of the rich households for different values for τ_R and δ_R, when $\delta_R = \delta_P$. The "S" region is for the equilibrium where the rich live in the suburbs, and the "G" region is for the equilibrium where the rich live in the central city.

Figure 9: Non-homothetic Preference for Amenities and Equilibrium Residential Patterns



Notes: Relative location of the rich households for different values for τ_R and δ_R, when $\delta_R > \delta_P$. The "S" region is for the equilibrium where the rich live in the suburbs, and the "G" region is for the equilibrium where the rich live in the central city. In the "I" region, the relative location of households is indeterminate.

To see the role of non-homothetic preference for amenities in affecting equilibrium residential patterns, I first set $\delta_R = \delta_P$, and check which type of equilibrium configuration arises for different combinations of τ_R and δ_R. Figure 8 displays the equilibrium configurations for different combinations of τ_R and δ_R. There is no indeterminacy in income-location relationship when $\delta_R = \delta_P$, depending on the size of τ_R, the rich either live in the city center or in the suburbs.

Next I set $\delta_R > \delta_P$, i.e., the rich benefit more from the amenities that they generate. Figure 9 shows the relationship between δ_R and the two cutoff values for τ_R within which the income-location relationship is indeterminate. When the income elasticity of commuting costs dominates, the equilibrium is always such that the rich in the city center. Because when it is too costly for the rich to commute, externalities and commuting costs both reinforce the rich's tendency to concentrate near the city center.

As shown in Figure 9, the cutoff value τ_1, below which the equilibrium is such that the rich in the suburbs, decreases with δ_R. If the rich benefit a lot from externalities, the "suburbanized equilibrium", where the rich live in the low-density suburbs, is less likely to arise. The "gentrified equilibrium" is a more preferable configuration with higher δ_R, because externalities are higher when the rich form high-density neighborhoods near the city center, than when they are scattered in low-density suburbs. This result implies that for externalities to have effects on the residential patterns within a city, the rich have to benefit more from neighborhood externalities than the poor do.

5 Conclusion

This chapter explores the effects of neighborhood amenities on equilibrium residential patterns in a spatial model of a city. Depending on the interactions between commuting costs and preferences for neighborhood amenities, the model is able to generate different equilibrium income-location relationships. In particular, when the impacts of externalities on rich households are high enough, the relative location of both income types is indeterminate. Such indeterminacy is absent in a standard monocentric model without externalities. From a theoretical point of view, this result is interesting in itself.

The results in this chapter demonstrate that the monocentric city model continues to be a useful framework to make sense of the recent trend of "central city revival". With commuting costs continue to fall over time, why high-income households are coming back to downtown areas remains a unresolved question in the monocentric framework. The model in this chapter provides one explanation for this puzzle. Concentration of jobs and scarcity of land lead to high population density near the city center, which endogenously facilitates consumption amenities that the rich prefer. Even it is not costly to commute from the suburbs, or if exogenous amenities in central cities no longer have much value, the rich would prefer central locations for better access to consumption.

In this chapter, endogenous amenities are simply modeled as being driven by the location choice of the rich, which, in turn have positive externalities on the utility of other households. It would be interesting to model the supply of amenities explicitly. The model does not distinguish different types of amenities. In reality, the nature and types of amenities differ for central and suburban neighborhoods. For example, entertainments and expensive consumption venues tend to locate near the city center; public services, like good schools, may be more common in suburban areas. How location-dependent amenities affect residential sorting in a city is an interesting issue to be explored in the future.

References

William Alonso et al. Location and land use. toward a general theory of land rent. *Location and land use. Toward a general theory of land rent.*, 1964.

Nathaniel Baum-Snow. Suburbanization and transportation in the monocentric model. *Journal of Urban Economics*, 62(3):405–423, 2007.

Leah Platt Boustan. Was postwar suburbanization “white flight”? evidence from the black migration. *The Quarterly Journal of Economics*, 125(1):417–443, 2010.

Victor Couture. Valuing the consumption benefits of urban density. *University of California, Berkeley. Processed*, 2013.

Victor Couture and Jessie Handbury. Urban revival in america, 2000 to 2010. Technical report, National Bureau of Economic Research, 2017.

Victor Couture, Cecile Gaubert, Jessie Handbury, and Erik Hurst. Income growth and the distributional effects of urban spatial sorting. Technical report, National Bureau of Economic Research, 2019.

Rebecca Diamond. The determinants and welfare implications of us workers' diverging location choices by skill: 1980-2000. *American Economic Review*, 106(3): 479–524, 2016.

Lena Edlund, Cecilia Machado, and Maria Micaela Sviatschi. Gentrification and the rising returns to skill. Technical report, National Bureau of Economic Research, 2015.

Ingrid Gould Ellen, Keren Mertens Horn, and Davin Reed. Has falling crime invited gentrification? *Journal of Housing Economics*, 46:101636, 2019.

Masahisa Fujita and Hideaki Ogawa. Multiple equilibria and structural transition of non-monocentric urban configurations. *Regional science and urban economics*, 12 (2):161–196, 1982.

Edward L Glaeser and Joshua D Gottlieb. Urban resurgence and the consumer city. *Urban studies*, 43(8):1275–1299, 2006.

Veronica Guerrieri, Daniel Hartley, and Erik Hurst. Endogenous gentrification and housing price dynamics. *Journal of Public Economics*, 100:45–60, 2013.

Jessie Handbury, Ilya Rahkovsky, and Molly Schnell. Is the focus on food deserts fruitless? retail access and food purchases across the socioeconomic spectrum. Technical report, National Bureau of Economic Research, 2015.

Sanghoon Lee and Jeffrey Lin. Natural amenities, neighbourhood dynamics, and persistence in the spatial distribution of income. *The Review of Economic Studies*, 85(1):663–694, 2018.

Edwin S Mills and Richard Price. Metropolitan suburbanization and central city problems. *Journal of Urban Economics*, 15(1):1–17, 1984.

Richard F Muth. Cities and housing; the spatial pattern of urban residential land use. 1969.

Yichen Su. The rising value of time and the origin of urban gentrification. 2018.

Appendices

In this appendix, I prove Proposition 1, and derive the equilibrium conditions for the "gentrified Equilibrium" where the rich live in the city center.

A Proof for the Proposition 1

Preferences over consumption, house size and location are given by

$$(1-\beta)\ln c + \beta \ln h + \delta lnH(x)$$

The budget constraint is

$$c + P_s(x)h = y_s - \tau_s x, \ s \in \{R, P\}.$$

Utility optimization implies that

$$\frac{c(x)}{h(x)} = \frac{1-\beta}{\beta} P_s(x)$$

Plugging $c(x) = y_s - \tau_s x - p(x)h(x)$ into the utility function, and differentiating with respect to x gives

$$\beta \frac{1}{h(x)} \frac{dh(x)}{dx} - (1-\beta)\frac{1}{c(x)} P_s(x) \frac{dh(x)}{dx} =$$
$$(1-\beta)\frac{1}{c(x)} \left[\frac{T(y_s, x)}{dx} + h(x)\frac{dP_s(x)}{dx} \right] - \delta_s \beta \frac{1}{E(x)} \frac{dE(x)}{dx}.$$

The first two terms on the left hand side cancel out, so the above expression becomes

$$\begin{aligned} \frac{dP_s(x)}{dx} &= \delta_s \frac{E'(x)}{E(x)} \frac{c_s(x)}{h_s(x)} \frac{1}{1-\beta} - \frac{\tau_s}{h_s(x)} \\ &= \delta_s \frac{E'(x)}{E(x)} \frac{P_s(x)}{\beta} - \frac{\tau_s}{h_s(x)}. \end{aligned}$$

B Sufficient Condition for the "Gentrified Equilibrium"

When the proposed equilibrium is the rich in the center, the rich's willingness to pay should be higher than that of the poor near the city center. We need to have

$$P_R(x) \geq P_P(x) \quad \text{for all } x \in (0, z],$$
$$P_R(x) < P_P(x) \quad \text{for all } x \in (z, b].$$

Recall that

$$P_s(x) = (1-\beta)^{\frac{1-\beta}{\beta}} \beta (\frac{1}{u_s})^{\frac{1}{\beta}} [y_s - \tau_s]^{\frac{1}{\beta}} E(x)^{\delta_s},$$

it is easy to check that

$$\frac{dP_s(x)}{dx} < 0.$$

So a necessary and sufficient condition for the solution to be an equilibrium is $P_R(x)$ is steeper than $P_P(x)$, which requires

$$\left.\frac{dP_R(x)}{dx}\right|_{x=z} < \left.\frac{dP_P(x)}{dx}\right|_{x=z}.$$

Evaluating $\frac{P_s(x)}{dx}$ at z gives

$$\delta_R \left.\frac{E'(x)}{E(x)}\right|_{x=z} \frac{P_R(z)}{\beta} - \frac{\tau_R}{h_R(z)} < \delta_P \left.\frac{E'(x)}{E(x)}\right|_{x=z} \frac{P_P(z)}{\beta} - \frac{\tau_P}{h_P(z)}. \tag{A1.1}$$

Notice that when $\delta_R = \delta_P$, this condition becomes

$$-\frac{\tau_R}{h_R(z)} < -\frac{\tau_P}{h_P(z)},$$

which is the same condition for the rich to live in the center as in the monocentric model without externality. This condition says that the rich live near the city center, when income elasticity of commuting cost is greater than income elasticity of housing demand.

When $\delta_R > \delta_P$, the equation (A1.1) can be arranged as

$$(\delta_R - \delta_P) \left.\frac{E'(x)}{E(x)}\right|_{x=z} \frac{P(z)}{\beta} < \frac{\tau_R}{h_R(z)} - \frac{\tau_P}{h_P(z)}.$$

After plugging in $h_s(x) = \beta \frac{y_s - \tau_R}{P_s(x)}$ and $\left.\frac{E'(x)}{E(x)}\right|_{x=z} = -\gamma$, this condition becomes

$$(\delta_R - \delta_P)(-\gamma) < \frac{\tau_R}{y_R - \tau_R z} - \frac{\tau_P}{y_P - \tau_P z}, \tag{B.1}$$

which always holds if $\frac{\tau_R}{y_R - \tau_R z} - \frac{\tau_P}{y_P - \tau_P z} > 0$. So a sufficient condition for the "gentrification equilibrium" to exist is

$$\frac{\tau_R}{y_R - \tau_R z} > \frac{\tau_P}{y_P - \tau_P z}.$$

C Sufficient Condition for the "Suburbanized Equilibrium"

When the proposed equilibrium is the rich in the center, the rich's willingness to pay should be higher than that of the poor near the city center. We need to have

$$P_R(x) < P_P(x) \quad \text{for all } x \in (0, z],$$
$$P_R(x) \geq P_P(x) \quad \text{for all } x \in (z, b],$$

with

$$P_s(x) = (1-\beta)^{\frac{1-\beta}{\beta}} \beta(\frac{1}{u_s})^{\frac{1}{\beta}} [y_s - T(y_s, x)]^{\frac{1}{\beta}} E(x)^{\delta_s}.$$

It is easy to check that a necessary condition for the solution to be an equilibrium is $P_R(x)$ is steeper than $P_P(x)$, which requires

$$\left.\frac{dP_R(x)}{dx}\right|_{x=z} > \left.\frac{dP_P(x)}{dx}\right|_{x=z}$$

Evaluating $\frac{P_s(x)}{dx}$ at z gives

$$\delta_R \left.\frac{E'(x)}{E(x)}\right|_{x=z} \frac{P_R(z)}{\beta} - \frac{\tau_R}{h_R(z)} > \delta_P \left.\frac{E'(x)}{E(x)}\right|_{x=z} \frac{P_P(z)}{\beta} - \frac{\tau_P}{h_P(z)}. \tag{A1.3}$$

Notice that when $\delta_R = \delta_P$, this condition becomes

$$-\frac{\tau_R}{h_R(z)} > -\frac{\tau_P}{h_P(z)},$$

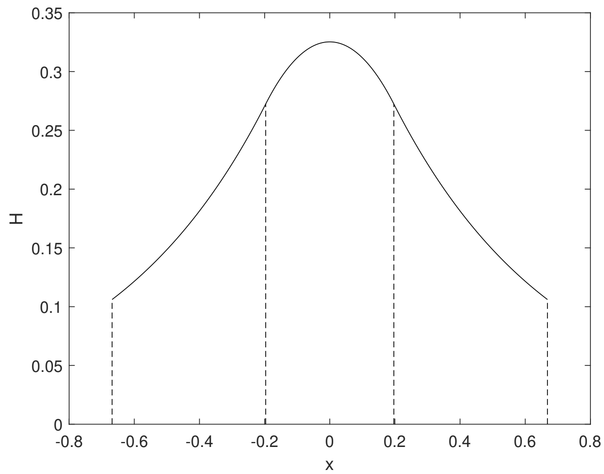
The condition for the rich to live in the periphery is exactly the same as the one in the standard monocentric model. When the income elasticity of housing demand is greater than the income elasticity of commuting cost, the rich would like to live in larger dwellings in the suburbs.

When $\delta_R > \delta_P$, the equation (A1.3) can be arranged as

$$(\delta_R - \delta_P)\,\gamma > \frac{\tau_R}{y_R - \tau_R z} - \frac{\tau_P}{y_P - \tau_P z}, \tag{A1.4}$$

which always holds if $\frac{\tau_R}{y_R - \tau_R z} < \frac{\tau_P}{y_P - \tau_P z}$.

Figure 10: The "Gentrified Equilibrium": Amenity Levels



Note: In this example, the boundary for the two neighborhoods is $z = 0.1433$*, and city size* $b = 0.5296$

D Conditions for the "Gentrified Equilibrium"

When the rich live in $x \in (0, z)]$ and the poor live in $x \in [z, b]$, the externality function takes the following form

$$E(x) = \begin{cases} \frac{2 - e^{-\gamma(x+z)} - e^{-\gamma(z-x)}}{\gamma} & \text{for } x \in [0, z) \\ \\ \frac{e^{\gamma(z-x)} - e^{-\gamma(z+x)}}{\gamma} & \text{for } x \in [z, b] \end{cases} . \tag{A2.1}$$

Figure 10 shows the level of neighborhood externalities across locations under the equilibrium configuration where the rich concentrate near the center. Externalities reach the maximum at $x = 0$ and declines with distance.

At city boundary $x = b$, the rent price for land $R(b)$ should equal to the rent for alternative land use $\underline{R}$, that is

$$R_P(b) = K_P\left[y_P - T(y_R, b)\right]^{\frac{1}{\alpha\beta}} E(b)^{\frac{\delta_P}{\alpha}} = \underline{R}.$$

K_P, can be written as

$$K_P = \underline{R}\left[y_P - T(y_P, b)\right]^{-\frac{1}{\alpha\beta}} E(b)^{-\frac{\delta_P}{\alpha}}.$$

At z, the rent for both types must be the same

$$R_R(z) = R_P(z),$$

which is

$$K_R\left[y_R - T(y_R, z)\right]^{\frac{1}{\alpha\beta}} E(z)^{\frac{\delta_R}{\alpha}} = K_P\left[y_P - T(y_P, z)\right]^{\frac{1}{\alpha\beta}} E(z)^{\frac{\delta_P}{\alpha}}.$$

K_R can be solved as

$$\begin{aligned} K_R &= K_P\left[\frac{y_P - T(y_P, z)}{y_R - T(y_R, z)}\right]^{\frac{1}{\alpha\beta}} E(z)^{\frac{\delta_P - \delta_R}{\alpha}} \\ &= \underline{R}\left\{\frac{y_P - T(y_P, z)}{\left[y_R - T(y_R, z)\right]\left[y_P - T(y_P, b)\right]}\right\}^{\frac{1}{\alpha\beta}} E(b)^{-\frac{\delta_P}{\alpha}} E(z)^{\frac{\delta_P - \delta_R}{\alpha}}. \end{aligned}$$

Market clearing conditions can be written as

$$\int_0^z n_R(x)dx = \frac{1}{\alpha\beta} K_R \int_0^z [y_R - T(y_R, x)]^{\frac{1}{\alpha\beta}-1} E(x)^{\frac{\delta_R}{\alpha}} dx = \frac{1}{2} N_R.$$
$$\int_z^b n_P(x)dx = \frac{1}{\alpha\beta} K_P \int_z^b [y_P - T(y_P, x)]^{\frac{1}{\alpha\beta}-1} E(x)^{\frac{\delta_P}{\alpha}} dx = \frac{1}{2} N_P.$$

After obtaining z and b, I need to verify that the solution is indeed an equilibrium, by checking that the rich are willing to pay more than the poor to live in the rich neighborhood, and the poor have higher willingness to pay in the poor neighborhood. Specifically I need to check that

$$P_R(x) > P_P(x) \quad \text{for all } x \in (0, z],$$
$$P_R(x) \leq P_P(x) \quad \text{for all } x \in (z, b].$$